ICTMA Newsletter

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The International Community of Teachers of Mathematical Modelling and Applications.
www.ictma.net

The Community, through its membership, research and other activities, is recognised as "The International Study Group for Mathematical Modelling and Applications (ICTMA)" by its affiliation to the International Commission on Mathematical Instruction (ICMI).

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Contents

Please send future contributions to the editor by email <gloria.stillman@acu.edu.au>. The next Newsletter will be published in December, 2014. We are interested in your contributions to any of the current sections including project reports and problems.

1. International Executive Committee

Following the business meeting in Blumenau on July 19, 2013, the ICTMA Executive for 2013-2015 was confirmed as follows:

President

Associate Professor Gloria Stillman (Australia) – Newsletter Editor & Secretary

Elected Members

Dr Jill Brown (Australia) [Email: Jill.Brown@acu.edu.au]
Prof Gabriele Kaiser (Germany) [Email: gabriele.kaiser@uni-hamburg.de]
Prof Jinxing Xie (China) – Webmaster & List Serve Moderator [Email: jxie@math.tsinghua.edu.cn]

Co-opted Members

Prof Helen Doerr (USA) [Email: hmdoerr@syr.edu]
Prof Toshikazu Ikeda (Japan) – Registrar [Email: ikeda@ed.ynu.ac.jp]
Prof Pauline Vos (Norway) [Email: pauline.vos@uia.no]

Conference Organisers

Prof Maria Salett Biembengut (Brazil) [Email: mariasalettb@gmail.com]
Associate Prof Geoff Wake (UK) [Email: Geoffrey.Wake@nottingham.ac.uk

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2. Upcoming Conference – ICTMA 17

ICTMA-17
NOTTINGHAM 2015

17TH INTERNATIONAL CONFERENCE ON THE TEACHING OF MATHEMATICAL MODELLING AND APPLICATIONS (ICTMA17) 19-24 JULY 2015

Conference Theme: **Modelling Perspectives: Looking within and across boundaries.**

The Centre for Research in Mathematics Education (CRME), University of Nottingham, U.K., will host the 17th International Conference of the Community of Teachers of Mathematical Modelling and Applications. Associate Professor Geoff Wake is Conference Chair.

Organisation

The conference will be organised locally by CRME staff, with the Conference Chair being supported by Professors Hugh Burkhardt and Malcolm Swan (Director of CRME), University of Nottingham. They will be supported by others of the U.K mathematics education research community who have worked in the field of mathematical modelling and applications for many years including Profs. Julian Williams (University of Manchester) and Mike Savage (University of Leeds). This team of experienced researchers and educators will seek to involve a number of newly established researchers in accordance with building community capacity.

Scientific Program and Public Lecture

The Conference theme is *Modelling perspectives: Looking within and across boundaries*. It is intended to provide a stimulus to consider new approaches drawing on best practice from other related research in mathematics education and associated domains. Modelling is considered as having potential for interdisciplinary work that is required for effective problem solving in the world of work and more widely. In developing the scientific program it is proposed to take the opportunity to strengthen and build our community taking the view that mathematical modelling and applications has potential appeal to a wider constituency than currently within the field of mathematics education research. The program will include plenary lectures, research and theoretical paper presentations and working groups. One page abstracts for paper presentations will be due in March, 2015. As always, an edited book of selected refereed chapters based on papers presented at the conference will be published by Springer after the conference. See ICTMA website for links to more recent book details. Proposed chapters will be due by September 15, 2015. Only a selection of these will be published after a strict reviewing process to ensure a high standard of scholarship and scientific quality.

In addition, drawing on the strength of CRME staff and others in the UK who work in mathematics education design research, the organizing team will invite participants to actively participate in an 'exhibition' of teaching and learning materials that prioritise mathematical modelling and applications. In the UK there is currently concern over public awareness of, and interest in, mathematics and science. A number of high-profile figures are working hard to reach audiences of those who ordinarily are wary of these subjects. It is hoped to organise a public lecture involving such a figure during the conference to assist this endeavor and to celebrate the ICTMA community assembling in Nottingham.

Venue

The conference will be held in the excellent academic facilities of The University of Nottingham's University Park campus with a range of accommodation facilities for delegates on-site at a range of different costs. En-suite study bedrooms with full board will be available at a very reasonable price and are recommended. University Park is Nottingham's largest campus at 300 acres. Part of the University since 1929, the campus is widely regarded as one of the largest and most attractive in the country. It is conveniently located only two miles from the city centre.

Social Program

The social program will include an opening reception, an excursion to Chatsworth House and the conference dinner in Colwick Hall, a local stately home.

Travel to and from Conference

The University of Nottingham is a major UK university, set in an attractive campus on the outskirts of the City of Nottingham in the East Midlands region of England. It is easily accessible from both within the UK and internationally. The city is well-served by a number of airports with direct flights from many European cities and from other continents through London, Manchester and Birmingham. From the airports in these cities Nottingham can be reached by train in about 90 minutes – 2 hours depending on route and time of day. Close at hand, East Midlands Airport (EMA) is most convenient with the city centre being easily reached by bus service in about 30-45 minutes.

Conference Website

www.nottingham.ac.uk/ICTMA17

For Further Information*:*

Email the conference chair: Geoff Wake at Geoffrey.Wake@nottingham.ac.uk

To pre-register for announcement emails please send your details to ICTMA17Academic@nottingham.ac.uk

3. Brief News Items

3.1 PME Research Forum on Mathematical Modelling in School Education

Many ICTMA members attended PME 38/PME-NA 36 in Vancouver, Canada, July 15-20, 2014. A research forum on *Mathematical Modelling in School Education: Mathematical, Curricular, Cognitive, Instructional, and Teacher Education Perspectives* was organized by Jinfai Cai and University of Delaware colleagues John Pelesko and Michelle Crillo with the support of ICTMA members Marcelo Borba, Gloria Stillman, Gabriele Kaiser, Lyn English and Rita Borromeo Ferri. Speakers at the forum also included Geoff Wake, Vince Geiger and OhNam Kwon. Proposed follow-up activities include a small edited book including chapters based on forum presentations. Modelling and applications were also the focus of several of the accepted research papers that were presented (see Section 6 of this Newsletter) as well as for many short oral presentations.

Research Forum Members Discussing Future Plans

A few of the very large German Contingent at PME enjoying the sunshine on the excursion.

3.2 TSG at ICME 13

ICME-13 brings together researchers, teacher educators, practising teachers, mathematicians, and others interested in the field of mathematics education from all over the world to discuss the state of the art of research and practice in mathematics education. It will be held in Hamburg, Germany, from July 8-31, 2016 at the Congress Center Hamburg near Dammtor Station, and the University of Hamburg's main building and campus. The congress is held every four years under the auspices of ICMI (International Commission on Mathematical Instruction).

TSG21 at ICME 13 will focus on Mathematical Applications and Modelling in the Teaching and Learning of Mathematics. The TSG will function as a Mini-Symposium, which displays the state-of-the-art-discussion bringing in renowned experts and allowing new scholars to enter the scene. The co-chairs are:

Gloria Stillman (Australian Catholic University, Australia), email: gloria.stillman@ac.edu.au

Jussara Araújo (Federal University of Minas Gerais, Brazil), email: jussara@mat.ufmg.br

The team members are:

Toshikazu Ikeda (Yokohama National University, Japan), email: toshi@ynu.ac.jp

Morten Blomhøj (Roskilde University, Denmark), email: blomhoej@ruc.dk

The team member from the German speaking countries is:

Dominik Leiss (Leuphana University Lüneburg, Germany), email: leiss@leuphana.de

IPC liaison person: Georg Ekol (Kyambogo University, Uganda), email: gle1@sfu.ca

Please visit our TSG on the ICME 13 website at www.icme13.org for updates. There will be also be further announcements in the next ICTMA Newsletter.

4. The 3rd Edition of the Curriculum Document of SEFI's Mathematics Working Group

It is one of the main goals of SEFI's (European Society for Engineering Education) Mathematics Working Group (MWG) to provide orientation to those who have a professional interest in the mathematical education of engineers. The core document that serves this purpose is the group's curriculum document that intends to help in clarifying the goals of mathematics education and ways to achieve them. In September 2013, the third edition of this document called "A Framework for Mathematics Curricula in Engineering Education" appeared. This can be downloaded from the group's website at http://sefi.htw-aalen.de.

During the last decade, in several seminars of the MWG the topic of "higher-level learning goals" came up which go beyond the largely content-related learning outcomes specified in the second edition. It is the main intention of the third edition to make use of state-of-the-art educational research in mathematics didactics in order to base the document on such a specification of higher-level goals. For doing this, the concept of "mathematical competence" and "competencies" was chosen which was developed in the Danish KOM project headed by Mogens Niss. This concept points the view to essential aspects of what mathematical education should strive for within an engineering study course:

"*Mathematical competence* then means the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations where mathematics plays or could play a role. Necessary, but certainly not sufficient, prerequisites for mathematical competence are lots of factual knowledge and technical skills, in the same way as vocabulary, orthography, and grammar are necessary but not sufficient prerequisites for literacy" (Niss, 2003)

This concept was adopted for the third edition of the MWG curriculum document mainly for two reasons. On the one hand, it emphasises the ability to apply mathematical concepts and procedures in relevant contexts which is the essential goal of mathematics in engineering education: to help students to work with engineering models and solve engineering problems. On the other hand, it explicitly recognises that competence requires a solid base of knowledge and skills reflecting the strong opinion of many "practitioners" engaged in the MWG. The concept is also well in line with current trends in general engineering education where the notion of competence has been used to describe educational goals which favour "action-based knowledge over knowledge simply held, in the name of performance and effectiveness" (Lemaitre et al., 2006).

In order to be helpful for curriculum specification the competence concept must be filled with more meaning. This has been done in the KOM project by identifying eight so-called competencies. The third edition of the MWG curriculum uses a slightly modified list of these competencies which are described in the second chapter of the document along with the three dimensions for specifying progress. The document does not prescribe a certain level of progress in the three dimensions since engineering study courses and engineering work profiles are much too heterogeneous for such an endeavour. Therefore, the third edition is called "A Framework for Mathematics Curricula in Engineering Education". Within this framework many curricula for different types of study courses can be specified. A potential process for building such profiles is also outlined in the second chapter.

The third chapter of the document presents a slightly modified version of the content-related learning outcomes that formed the kernel of the second edition. The clear arrangement into four levels was retained. Although the authors think that most of the learning outcomes specified in core zero and core level 1 should still be covered in any engineering study course, the lists in chapter three are also offered as a framework from which a choice must be made based on the requirements of the study course. Since in many European countries the number of contact hours in mathematics has been reduced when introducing the bachelor-master split, one has to make a realistic choice regarding the learning outcomes.

The fourth chapter of the document discusses aspects of adequate teaching and learning environments in a competence-based engineering study course and provides many links to relevant former seminar contributions and other literature. Learning scenarios like lectures, projects, assignments, tutorials, laboratories and technology-enhanced settings are investigated for their potential for obtaining competencies. A recommendation for a mix of offerings is given. Next, transition problems are explained and successful measures for addressing these are outlined. The use of mathematics technology which has been a subject of controversial debate in many seminars is also of special interest in a competence-based approach. It is even explicitly addressed in the eighth competency on using aids and tools.

Enabling students to understand and use mathematics in engineering contexts forms the kernel of the competence concept. For this reason, a mathematics curriculum that seriously intends to support this concept must be strongly integrated into the engineering study course for which it has been set up. There are several aspects of this integration like "Which mathematics is used in application subjects and how is it used?" or "When are mathematical concepts needed (early and again in later study phases)?". These are discussed and some interesting approaches from literature are outlined. Strongly related to the question of integration is the attitude of students towards mathematics which is explained in the final section of the chapter. Is it seen as a stumbling block to overcome at the beginning or is it seen as integral part of engineering? Such attitudes strongly influence motivation and readiness to apply (or avoid) a mathematical approach to engineering problems and hence must be taken into account.

The fifth chapter of the document is concerned with assessment. Since many students are extremely assessment driven adequate assessment regimes must not be neglected. The chapter first describes different forms of assessment which are applied in Europe. It then discusses the question of requirements for passing which are essential for guaranteeing that after passing an examination students have really achieved the learning outcomes specified in a curriculum. Whereas assessing very detailed, content-related learning outcomes is often straightforward, the assessment of competencies is more challenging and still the topic of current research. Finally, aspects of technology-supported assessment are discussed.

The document is not meant to be a "handbook" for the mathematical education of engineers but it aims at providing orientation and hints regarding essential topics. The group's website provides more information on the curriculum document as well as on current activities and seminars of the group.

References

Lemaitre, D., Le Prat, R., De Graaff, E., & Bot, L. (2006). Editorial: Focusing on competence, *European Journal of Engineering Education, 31*(1), 47.

Niss, M. (2003). Mathematical competencies and the learning of mathematics: The Danish KOM project. In A. Gagatsis & S. Papastravidis, S. (Eds.), *Proceedings of the 3rd Mediterranean Conference on Mathematics Education*, Athens 2003, (pp. 115-124).

Burkhard Alpers, Aalen University of Applied Sciences, Germany

5. Recent Dissertations

Biccard, P. (2013). *The didactisation practices in primary school mathematics teachers through modelling.* Doctor of Philosophy thesis, Stellenbosch University, Supervisor DCJ Wessels.

Mathematics teacher development is a source of national and international concern. This study describes how primary school mathematics teachers develop didactisation practices. In considering how teachers could develop, so that student learning is optimised; the concepts of didactisation and the mathematical work of teaching were sourced from existing literature. The concept of *didactisation* is explored and defined; and is incorporated with the concept of *mathematical work of teaching*. Nine practices were made explicit through this incorporation: active students, differentiation, mathematisation, vertically aligned lessons, access, probe, connect and assess student thinking, and teacher reflection. These nine practices become the framework for the professional development program and the data generation structure. Five primary school teachers were involved in a professional development program that used model-eliciting activities (MEAs) as a point of departure. A modelling perspective to teacher learning was chosen for the professional development program. The methodology followed the principles of design research and from this, a three phase teaching experiment was designed and implemented. The teachers and researcher met for development sessions and teachers were observed in practice at intervals throughout the program. Their developing didactisation practices were documented through a qualitative analysis of the data. It was established that teachers' didactisation practices did develop during the nine-month program. Furthermore it was found that didactisation practices developed at different rates and consequently, a hierarchy of didactisation practice development is presented. The impact of the program was also gauged through teachers' changing resources, goals and orientations. These three aspects also evolved over time. The program proposed in this study may be a suitable model to develop in-service and pre-service mathematics teachers. The study contributes to understanding teacher action in a classroom and how teachers can change their own thinking and practice.

Czocher, J. (2013). Toward a description of how engineering students think mathematically. Doctor of Philosophy Dissertation. The Ohio State University. Supervisor: Azita Manoucheri [Available at: http://rave.ohiolink.edu/etdc/view?acc_num=osu1371873286]

The purpose of this study was to build a descriptive model for how individuals add mathematical structure to a problem setting. Blum and Leiß's (2007) mathematical modelling cycle was adopted as a research framework. Data were collected using task-based clinical interviews with four engineering students enrolled in a differential equations course. Analysis led to the creation of three theoretical constructs which together describe the process of structurally enriching (Schwarzkopf, 2007) a non-mathematical context: mathematical framing, the pseudo-empirical setting, and intertwining. The students in this study made sense of the modelling tasks by assuming that the solution had a certain mathematical structure (*the mathematical framing*) and then verifying that choice by making sure all the necessary information was present (comparing the information in the task to *the pseudo-empirical setting*). This process of matching up the variables and operations in the mathematical framing with the quantities and relationships in the pseudo-empirical setting was called *intertwining*. To move forward in the modelling task, the students then externalized a mathematical representation and analyzed it. Validating activity was observed throughout this process and analysis confirmed five distinct types of validating activity: (i) to check alignment of the mathematical representation with the individual's interpretation of the context (checking mathematical representation against the real model); (ii) confirming alignment between the mathematical framing and the individual's interpretation of the context (checking the mathematical representation against the situation model); (iii) to check alignment between the results of analysis and the individual's interpretation of the context (checking the real results against the real model); (iv) to check the analysis itself (checking mathematical results against the mathematical representation); (v) to check agreement between the results of the analysis and the information available from the real world (checking real results against the situation model).

These findings were used to build a theoretical model of the mathematical modelling process which deviates from previous theoretical and research frameworks used to study mathematical modelling.

P. Frejd (2014). Modes of Mathematical Modelling: An analysis of how modelling is used and interpreted in and out of school settings. Supervisors: C. Bergsten & J. B. Ärlebäck. Linköping University.

The relevance of using mathematics in and for out-of-school activities is one main argument for teaching mathematics in education. Mathematical modelling is considered as a bridge between the mathematics learned and taught in schools and the mathematics used at the workplace and in society and it is also a central notion in the present Swedish mathematical syllabus for upper secondary school. This doctoral thesis reports on students', teachers' and modelling experts' experiences of, learning, teaching and working with mathematical modelling in and out of school settings and their interpretations of the notion of mathematical modelling.

The thesis includes five papers and a preamble, where the papers are summarised, analysed, and discussed. Different methods are being used in the thesis such as video analysis of students' collaboration working with a modelling problem, interview investigations with teachers and expert modellers, content analysis of textbooks and a literature review of modelling assessment. Theoretical aspects concerning mathematical modelling and the didactic transposition of modelling are examined.

The results presented in this thesis provide a fragmented picture of the didactic transposition of mathematical modelling in school mathematics in Sweden. There are significant differences in how modellers, teachers and students work with modelling in different practices in terms of the goal of the modelling activity, the risks involved in using the models, the use of technology, division of labour and the construction of mathematical models. However, there are also similarities identified and described as important aspects of modelling work in the different practices, such as communication, collaboration, projects, and the use of applying and adapting pre-defined models. Students, teachers and modellers expressed a variety of descriptions of what modelling means. The variety of descriptions in the workplace is not surprising, since their working approaches are quite different, but it makes the notion difficult to transpose into school practice. Questions raised are if it is unrealistic to search for a general definition and if it is really necessary to have a general definition. The consequence, for anyone who uses the notion, is to always be explicit with the meaning.

An implication for teaching is that modelling as it shows in the workplace can never be fully 'mapped' in the mathematical classroom. However, it may be possible to 'simulate' such activity. Working with mathematical modelling in projects is suggested to simulate workplace activities, which include collaboration and communication between different participants. The modelling problems may for example involve economic and environmental decisions, to prepare students to be critically aware of the use of mathematics in private life and in society, where many decisions are based on mathematical models.

Van Buuren, O. (2014). Development of a modeling learning path. Dissertation. University of Amsterdam. Supervisors: A. L. Ellermeijer & A. J. P. Heck [Available at: http://dare.uva.nl/record/471193]

In this dissertation, a design research project is reported on in which a learning path on computational modelling, integrated into the Dutch lower secondary physics curriculum, has been developed and tested in school practice. The instructional materials that were developed cover the first two years of this curriculum (starting from ages 13-14). The research questions addressed included: What are characteristics of an effective learning path on graphical modelling in lower secondary education? To what extent do students learn to model when they follow this learning path?

In the learning path, emphasis was placed on modelling with computers. System dynamics based graphical modelling was chosen as the modelling approach where modelling was systematically combined with experimenting and doing measurements. The experiments familiarize students with the realistic situations that are modeled; the measurements provide the data that are used for evaluation and validation of the models. Computer modelling also enables students to study subjects that are more realistic and thus more complex. The graphical version of Forrester's system dynamics was used. In this approach, model equations are represented by a graphical diagram consisting of a structure of icons. Blockages as a result of limited student notions of variable and formula, that were detected in a pilot study, were able to be overcome by offering students operational definitions of variable and formula, by letting students use formulas in a computer learning environment, and by letting students construct simple formulas themselves. The progress in the students' abilities to construct simple formulas is promising when compared to results of modelling in upper secondary education. Important results of this developmental research project are the design principles that have evolved in the course of this project. Consequences of integration of modelling into the physics curriculum are discussed. It is shown that modelling requires a higher level of conceptual mathematical understanding than usual in physics education, but it is also shown how this higher level of understanding can be achieved by students. Students successfully worked with several graphical models and successfully constructed simple graphical models based on known equations, but only some of the students correctly understood all aspects of the graphical diagrams. Reality-based interpretation of the graphical diagrams can conceal an incorrect understanding of diagram structures. As a result, students seemingly have no problems interpreting these diagrams until they are asked to construct a graphical model without assistance. The model equations are not communicated clearly enough by the graphical diagrams. Despite this, at the end of the learning path, students who had followed this path were able to build simple models without teacher assistance. Modelling takes time, but this time may be well spent, because of the value of modelling as a general competency, that is useful for other disciplines as well. This research project has shown that it is possible to start with modelling in lower secondary education, provided that sufficient attention is paid to modelling-related student difficulties. The learning path is to be extended into upper secondary education in the future.

6. Recent Publications of Interest

Bahmaei, F. (2014). Mathematical modelling in primary school, advantages and challenges. *Journal of Mathematical Modelling and Applications, 1*(9), 3-13.

Barabash, M., Guberman, R., & Mandler, D. (2014). Primary school teachers learn modelling: How deep should their mathematics knowledge be? In C. Nicol, P. Liljedahl, S. Oesterle, & D. Allan (Eds.), *Proceedings of the 38th Conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics Education* (Vol. 2, pp. 89-96). Vancouver, Canada: PME.

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Chan, C. M. E. (2014). Exploring group dynamics of Primary 6 students engaged in mathematical modelling activities. In J. Anderson, M. Cavanagh & A. Prescott (Eds.), *Curriculum in focus: Research guided practice* (Proceedings of the Thirty-seventh annual MERGA conference, Sydney, pp. 127-132). Adelaide: AAMT/MERGA. [Available from: www.merga.net.au]

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Lamb, J., Kawakami, T., & Matsuzaki, A. (2014). Leading a new pedagogical approach to Australian curriculum mathematics: Using the dual modelling cycle framework. In J. Anderson, M. Cavanagh & A. Prescott (Eds.), *Curriculum in focus: Research guided practice* (Proceedings of the Thirty-seventh annual MERGA conference, Sydney, pp. 357-364). Adelaide: AAMT/MERGA. [Available from: www.merga.net.au]
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Vorhölter, K., Kaiser, G., & Borromeo Ferri, R. (2014). Modelling in mathematics classroom instruction: An innovative approach for transforming mathematics education. In Y. Li et al. (Eds.), *Transforming mathematics education: Multiple approaches and practices* (pp. 21-36). Cham, Switzerland: Springer.
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Wedelin, D., & Adawi, T. (2014). Teaching mathematical modelling and problem solving: A cognitive apprenticeship approach to mathematics and engineering education. *International Journal of Engineering Pedagogy, 4*(5), 49-55.
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7. Modelling Problems

At the business meeting in Blumenau it was suggested by members that we start a modelling problem section in the Newsletter for members to work on across the world. I am now calling for such problems to be contributed for distribution via the List Serve with solutions, comments, progress (as appropriate) published in the next Newsletter in June 2014. Please send your suggestion to the Editor so it can be placed on the Listserve for members to contribute. The following is the first contribution received fro this section.

Remarks on and Examples of Mathematical Modelling Problems

Pollak (1979) was one of the first who described the process of Mathematical Modelling (MM) in a way that could be used in teaching mathematics (Circle of Modelling: ICME-3, Karlsruhe, 1976). Since then much effort has been placed by researchers and educators to analyze in detail the MM process. A brief but comprehensive account of the different models used for this purpose can be found in Haines & Crouch (2010), including my stochastic model (Voskoglou, 2007). With this model we have treated the MM circle as a Markov chain process dependent upon the transition between the successive discrete stages of the MM process.

Models for the MM process like the above are useful in understanding what is termed in Haines and Crouch (2010) as the '*ideal behaviour'*, in which the modellers proceed effortlessly from a real world problem through a mathematical model to acceptable solutions and report on them. However, life in classrooms (and amongst modellers in industry and elsewhere as well) is not like that. More recent research (Borromeo Ferri, 2007; Doerr, 2007; Gailbraith & Stillman, 2001 etc) reports that students in school take *individual routes* when tackling MM problems, associated with their individual learning styles and the level of their cognition, which utilizes in general concepts that are inherently graded and therefore fuzzy. On the other hand, from the teachers' point of view there usually exists a degree of vagueness about their students' way of thinking in each of the stages of the MM process, when tackling such kind of problems. All these gave us the impulsion to introduce principles of *fuzzy logic* for treating in a more realistic way the process of MM in the classroom, and to use also the concept of *uncertainty* - which emerges naturally within the broad framework of fuzzy sets theory- in obtaining a measure of the students MM skills (Voskoglou, 2010).

As a result of all the research efforts mentioned above it is more or less accepted that the process of MM in the classroom basically involves the following stages: *Analysis* of the problem, *mathematization*, *solution* of the model, *validation* (control) of the model and *implementation* of the final mathematical results to the real system (e.g., see Voskoglou, 2007 or 2010). One may notice that some authors consider further stages in the MM process; for example, some of them divide mathematization to the stages of the *formulation* of the real problem in a way that it will be ready for mathematical treatment and of the *construction* of the model, others divide validation into the stages of *interpretation* and *evaluation* of the model or/and they add the stage of *refining* the model, etcetera (e.g., see Haines & Crouch, 2010). However, all these minor variations do not change the general idea that we nowadays have about the circle of MM in the classroom.

There is no doubt that mathematization possesses the greatest gravity among all stages of the MM process, since it involves a deep abstracting process, which is not always an easy thing to be achieved by a non-expert. However, as Crouch and Haines (2004, section 1) report, it is the interface between the real world problem and the mathematical model that presents difficulties to the students, that is, the transition from the real word to the mathematical model (i.e. the mathematization) and *vice versa the transition from the solution of the model to the real world*. The latter looks rather surprising at first glance, since, at least for the type of MM problems usually solved at secondary schools, a student who has obtained a mathematical solution of the model is normally expected to be able to "translate" it easily in terms of the corresponding real situation and to check its validity. However, things are not always like that. In fact, there are sometimes MM situations, where the validation of the model and/or the implementation of the final mathematical results to the real system, hide surprises that force students to "look back" to the construction of the model making (possibly) the necessary changes to it.

The following two examples (problems), derived from our students' reactions in the Graduate Technological Educational Institute (T.E.I.) of Western Greece attending the course "Higher Mathematics I" of their first term of studies, illustrate in a good way such kind of situations.

Problem 1: We want to construct a channel to run water by folding the two edges of an orthogonal metallic leaf having sides of length *20 cm* and *32 cm*, in such a way that they will be perpendicular to the other parts of the leaf. Assuming that the flow of the water is constant, how can we run the maximum possible quantity of the water?

Solution: Folding the two edges of the metallic leaf by length x across its longer side the vertical cut of the constructed channel forms an orthogonal with sides x and $32-2x$ (Figure 1).

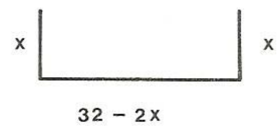

Fig. 1: The vertical cut of the channel

The area of the orthogonal, which is equal to $E(x) = x(32-2x) = 32x-2x^2$, has to be maximized. Taking the derivative $E'(x)$ the equation $E'(x) = 32-4x = 0$ gives that $x = 8$ *cm.* But $E''(x) = -4 < 0$, therefore $E(8) = 128\ cm^2$ is the maximum possible quantity of water to run through the channel.

Remark: A number of students folded the edges of the other side of the leaf and they found that $E(x) = x(20-2x) = 20x-2x^2$. In this case the equation $E'(x) = 0$ gives that $x = 5$ *cm*, while $E(5) = 50\ cm^2$. Their solution was of course mathematically correct, but many of them failed to realize that it is not acceptable in practice (real world).

Problem 2: Among all the cylindrical towers having a total surface of 180π m^2, which one has the maximal volume?

Solution: Let R be the radius of the basement of the tower and let h be its height. Then its total surface is equal to $2\pi Rh+2\pi R^2=180\pi \Rightarrow h = \frac{90-R^2}{R}$. Therefore the volume of the tower as a function of R is equal to $V(R) = \pi R^2 \frac{90-R^2}{R}=90\pi R-\pi R^3$. But $V'(R) = 90\pi-3\pi R^2 = 0$ gives that $R = \sqrt{30}$ *m, while* $V''(R) = -6\pi R < 0$. Thus the maximal volume of the tower is equal to $V(\sqrt{30}) =90\pi\sqrt{30}-\pi(\sqrt{30})^3 = 60\sqrt{30}\pi \approx 1032\ m^3$

Remark: A number of students considered the total surface of the tower as being equal to $2\pi Rh$, not including to it the areas of its basement and its roof. In this case they found that $h = \frac{90}{R}$, $V(R) = 90\pi R$ *and* $V'(R) = 90\pi > 0$, which means that under these conditions there is no tower having a maximal volume. However, some of these students failed to correct their model in order to find the existing solution of the real problem (unsuccessful transition from the model to the real world).

Examples like the two presented above give to the teacher an excellent opportunity to discuss in the class all of their students' reactions (correct and wrong), thus emphasizing the importance of the last two stages of the MM process (validation and implementation of the model) in solving real world problems.

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Comment on Problem 1

In the workplace, any product with bends in it will require more material to achieve the final design shape than a flat shape of the same size. The angle of the bend also contributes to this. The impact for producing one product is minimal, but when producing thousands the impact is significantly increased. Therefore, more material will be required.