Sensemaking in model-eliciting activities: what can we learn from other domains?

ICTMA 18 - Cape Town, 23-28 July 2017
Piera Biccard: University of South Africa

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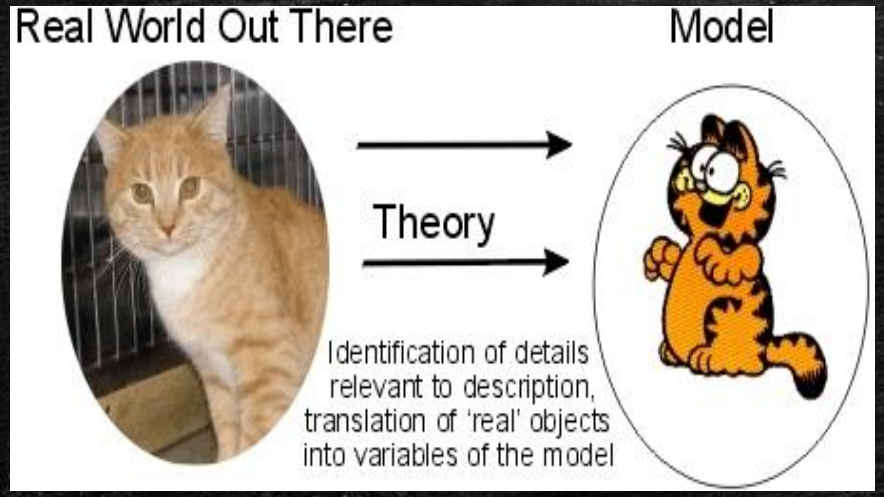
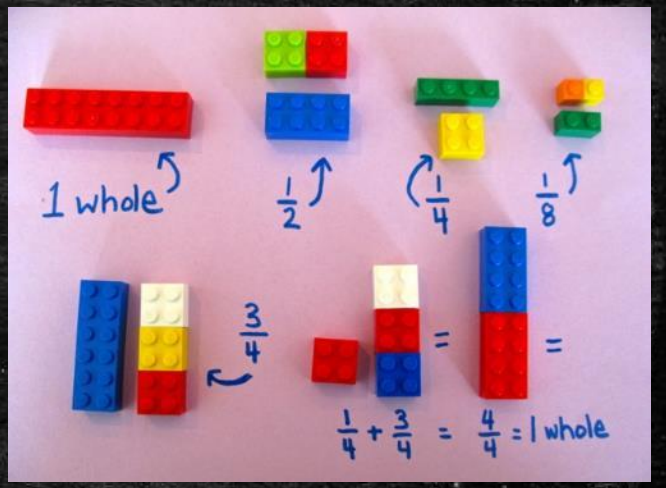
Models?



http://backreaction.blogspot.co.za/2008/04/models-and-theories.html

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

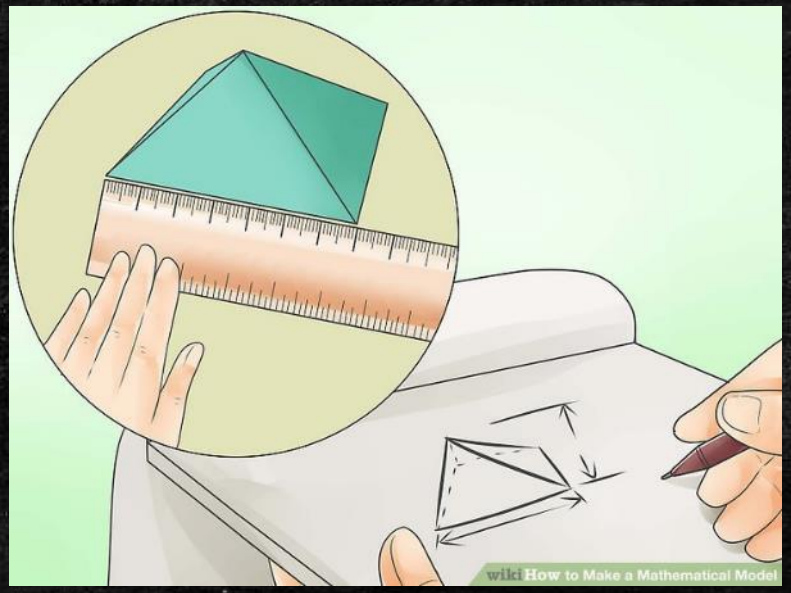
http://www.mathematics-monster.com/lessons/quadratic_equations_formula.html

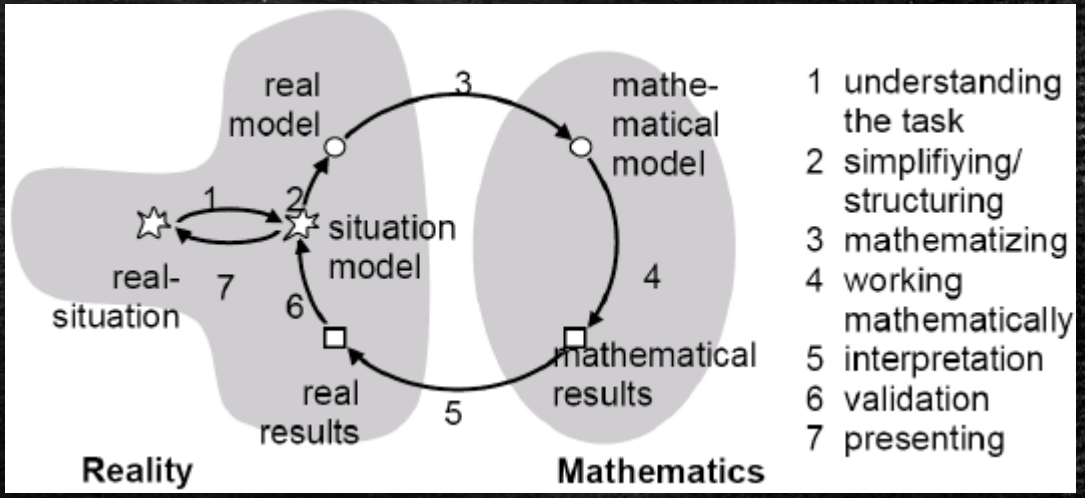


https://www.scholastic.com/teachers/blog-posts/alycia-zimmerman/using-lego-build-math-concepts/

Modelling?

Images: Pixabay





Blum & Leiss, 2006

Lesh and Doerr (2003 p. 10) define models as "conceptual systems that consist of elements, relations, operations and rules governing interactions that are expressed using external notations system and that are used to construct, describe, or explain the behaviours of other systems - perhaps so that the other system can be manipulated or predicted intelligently. A mathematical model focuses on structural characteristics (rather than for example physical or musical characteristics) of relevant systems."

Stachowiak, 1973

A mapping feature - a model is based on an original

A reduction feature - a model only reflects a selection of the original's properties

A pragmatic feature - it needs to be usable in the place of the original

Discovering relationships precedes finding variables to describe them

We present students with a system (real world)

3 year lag?
(Treilibs et al.)

We don't even know what mathematics they will resort to

Modelling (model-eliciting activities)

We ask them to create a model to tell us something about the system (our question)

They do not (necessarily) apply a known method

They do not (necessarily) follow known procedures

"Any mathematical experience in which students make choices about how to use mathematics to create representations of a real-world process is a form of mathematical modelling"

Gann et al. 2016

"learning to **make decisions** and assumptions and on using mathematics to understand a real-world-scenario". (Gann et al., 2016 p. 105)

Treilibs et al. 1980. Poor modellers avoid making decisions of any sort entirely

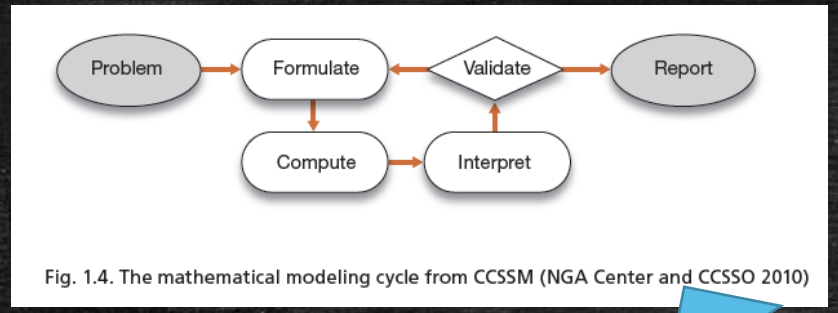
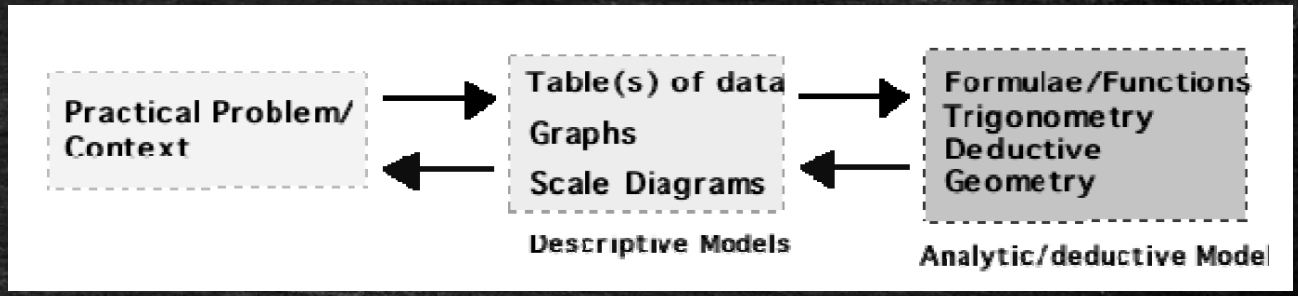


Fig. 1.4. The mathematical modeling cycle from CCSSM (NGA Center and CCSSO 2010)



(De Villiers 2007, 58)

Cirillo, Pelesko, Felton-Koestler & Rubel, 2016

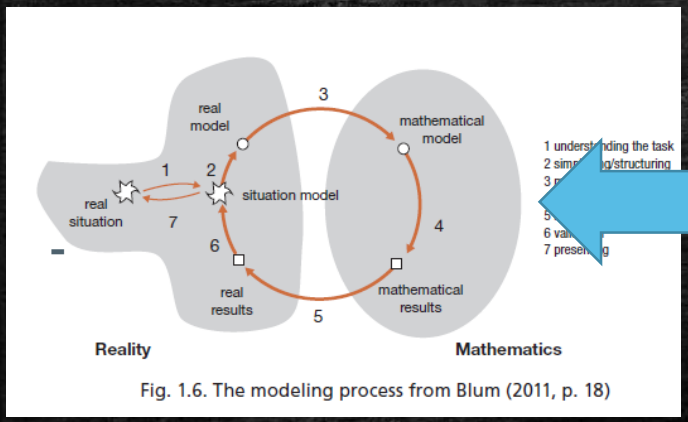


Fig. 1.6. The modeling process from Blum (2011, p. 18)

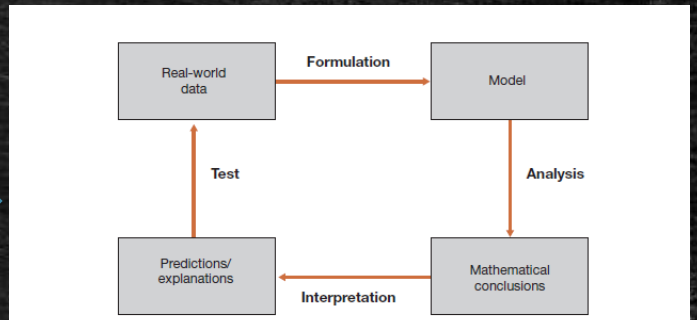


Fig. 1.5. The modeling process portrayed as a closed system (Dossey et al. 2002, p. 114)

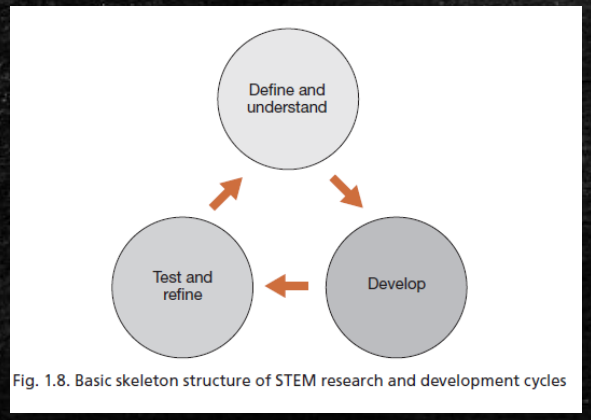


Fig. 1.8. Basic skeleton structure of STEM research and development cycles

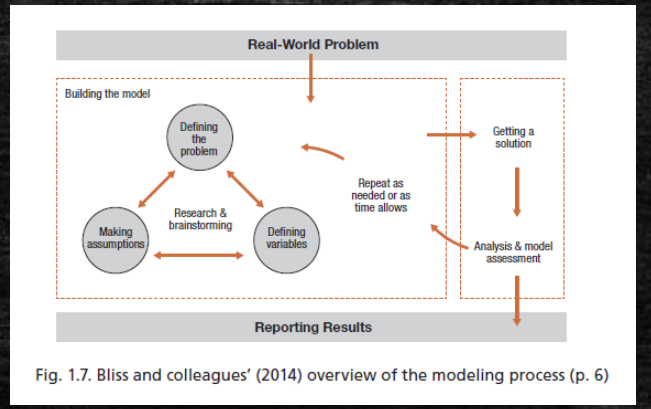


Fig. 1.7. Bliss and colleagues' (2014) overview of the modeling process (p. 6)

Cirillo, Pelesko, Felton-Koestler & Rubel, 2016

Mathematical modelling

vs

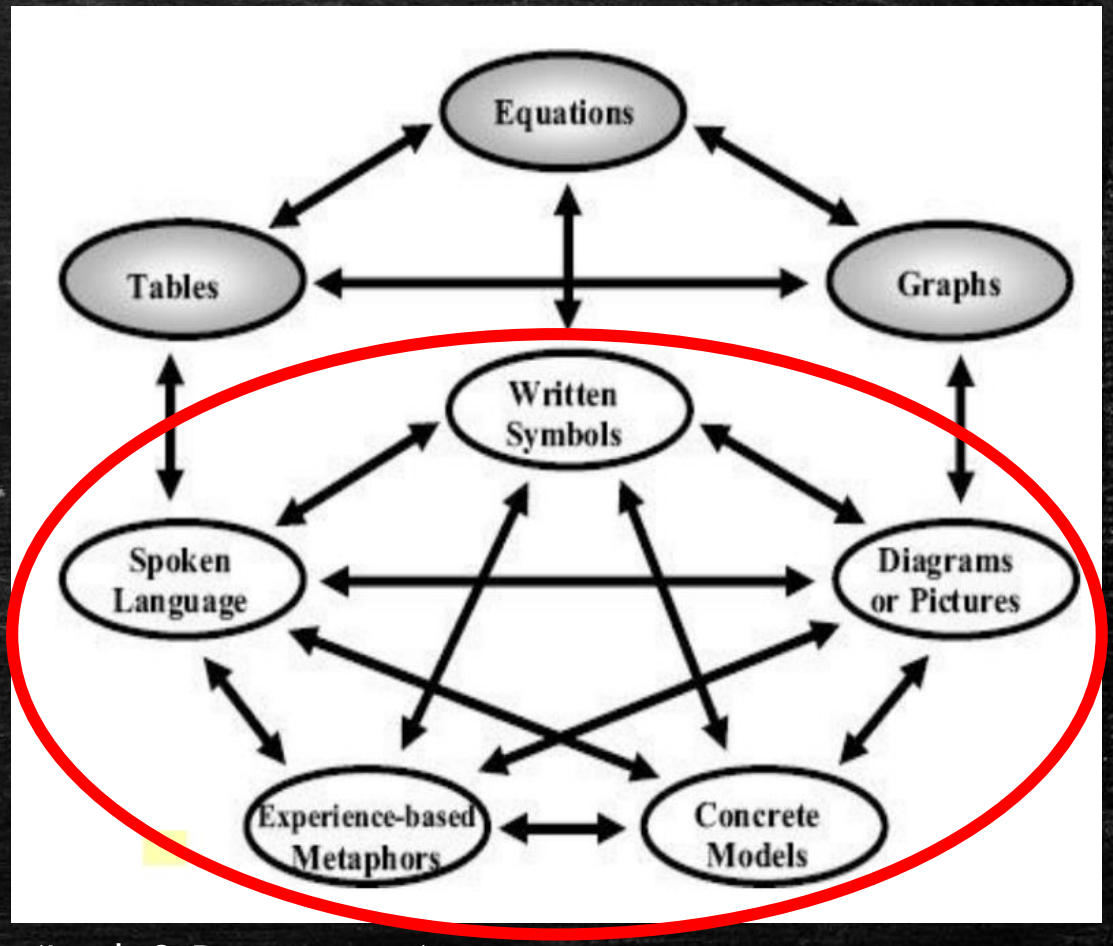
Modelling mathematics

Modelling mathematics

(Noun based)

Mathematics

Look for something in reality that represents it

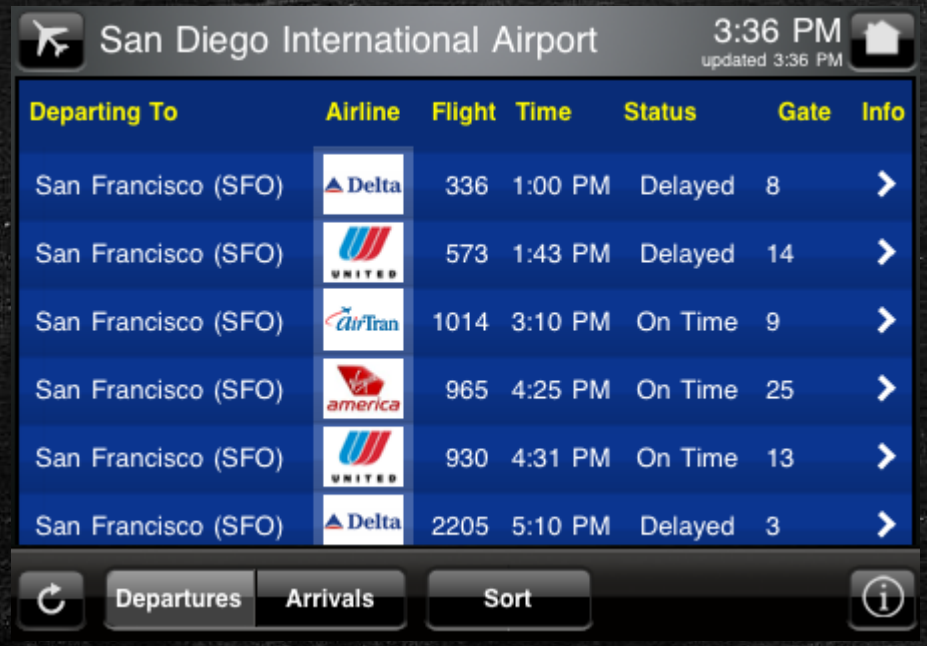


(Lesh & Doerr, 2003)

Mathematical modelling

Messy, real problem...

Verb based 'to model'



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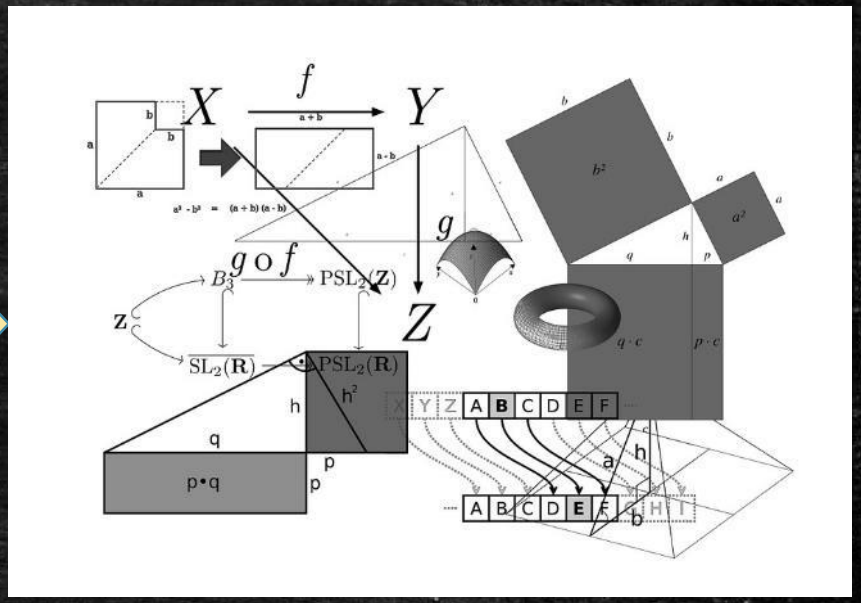
Look for something in mathematics to help structure it



Commons.Wikipedia.org

Mathematical 'world'

Tam (in Groshong, 2016)

First learn how to model mathematics before students create their own models

English, 2003

Young learners (who do not have ready-made mathematical tools) can be involved in modelling

Hamilton, 2007

Students will invent their own version of tools because they have decided that these tools are necessary

In-authentic

Authentic

Eric Mazur, 2014. Assessment, the silent killer of learning.

Students have to be involved in sensemaking

Students have to practice sensemaking

Stender & Kaiser, 2016
Modelling competencies can only develop when students actually work on complex modelling problems

(Bliss & Libertine, 2016)

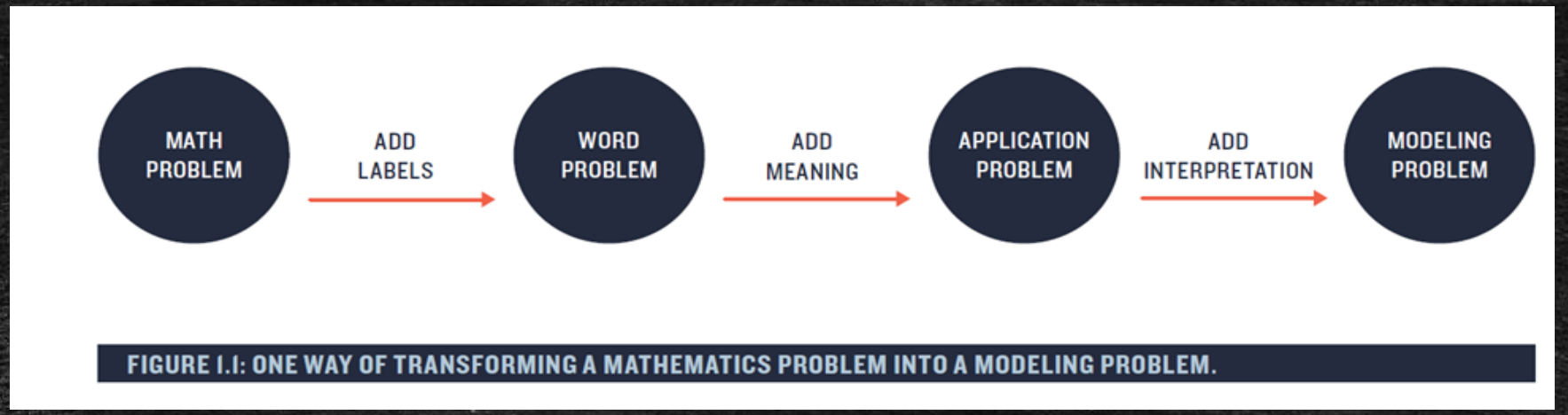


FIGURE I.I: ONE WAY OF TRANSFORMING A MATHEMATICS PROBLEM INTO A MODELING PROBLEM.

STRUCTURING/ FEWER DECISIONS

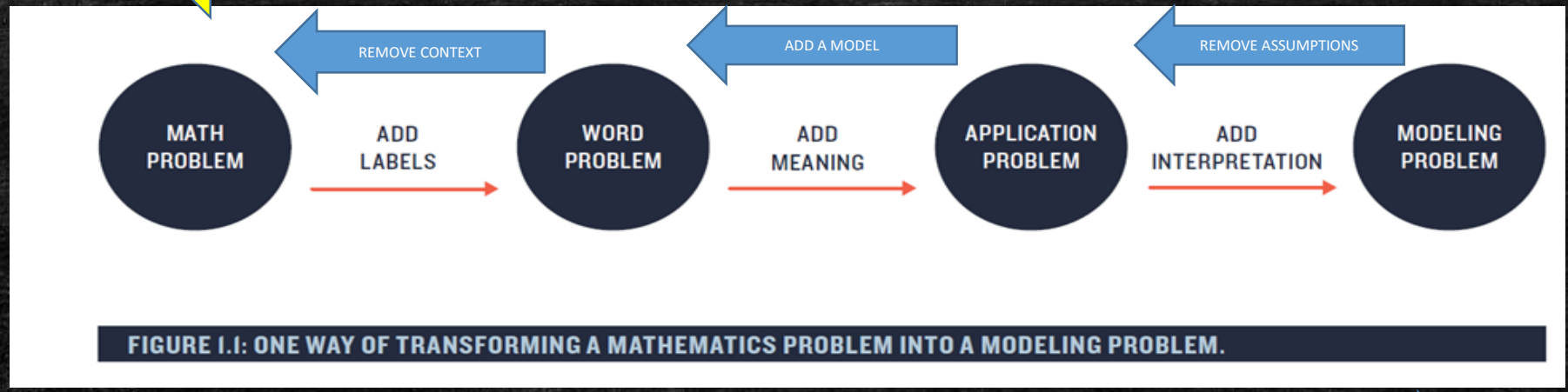


FIGURE I.I: ONE WAY OF TRANSFORMING A MATHEMATICS PROBLEM INTO A MODELING PROBLEM.

SENSEMAKING/MORE DECISIONS

Sensemaking

www.pixabay.com

Deepening understanding

(van Velzen, 2016)

Application of concepts

Conceptual vs procedural thinking

Advanced mathematical thinking

Instrumental vs relational thinking

"developing understanding of a situation, context, or concept by connecting it with existing knowledge" (NCTM, 2009, p. 4)

Klein, Moon, and Hoffman (2006)

Sensemaking is the ability or attempt to make sense of an ambiguous situation. More exactly, sensemaking is the process of creating situational awareness and understanding in situations of high complexity or uncertainty in order to make decisions. It is a motivated, continuous effort to understand connections in order to anticipate their trajectories and act effectively.

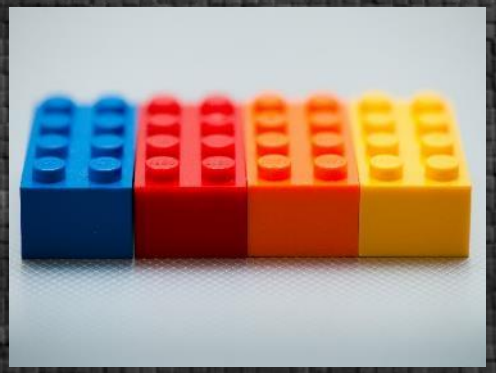
Myths

Klein, Hoffer, Moon (2006)

More information leads to better sensemaking

Clicker-free vector images

Data fusion assists sensemaking



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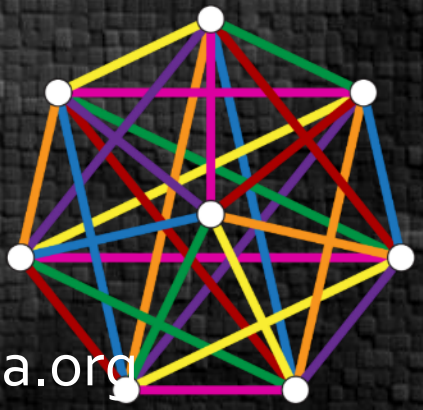
It is not simply about connecting the dots…



www.en.Wikipedia.org

Myths

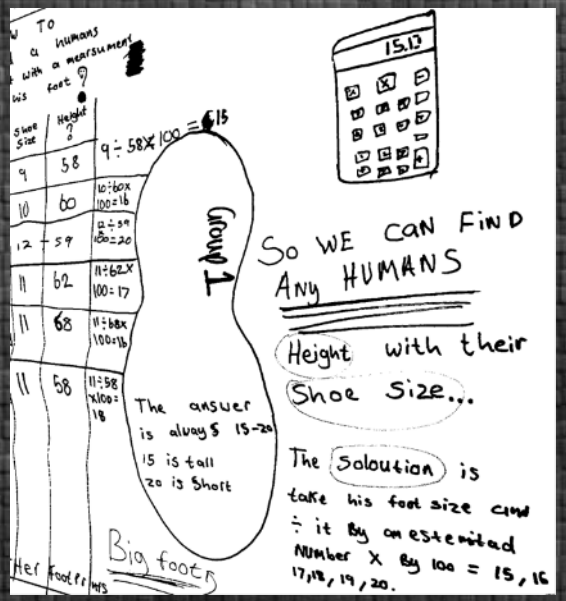
Keeping an open mind is important

Sensemaking follows the form of information-knowledge-understanding

Klein, Hoffer, Moon (2006)

novices

confirm



disconfirm

experts

Biccard, 2010

"process of learning whereby individual cases, or small, contextually bound understandings with specific inference possibilities attached to them, might develop into larger, more organized understandings" (Klein et al. 2006, p. 91)

Savolainen (1993, p. 16)

Sense making is the process….sense is the product

Dervin (in Savolainen)

Sense includes not only knowledge

Individual's interpretations

intuitions

responses

opinions

questions

evaluations

hunches

Weick (1995)

"sense-making is about the placement of items into frameworks, comprehending, redressing surprises, constructing meaning, interacting in the pursuit of mutual understanding, and patterning· It is grounded in both individual and social activity"

"Developing set of ideas with explanatory possibilities rather than as a body of knowledge"

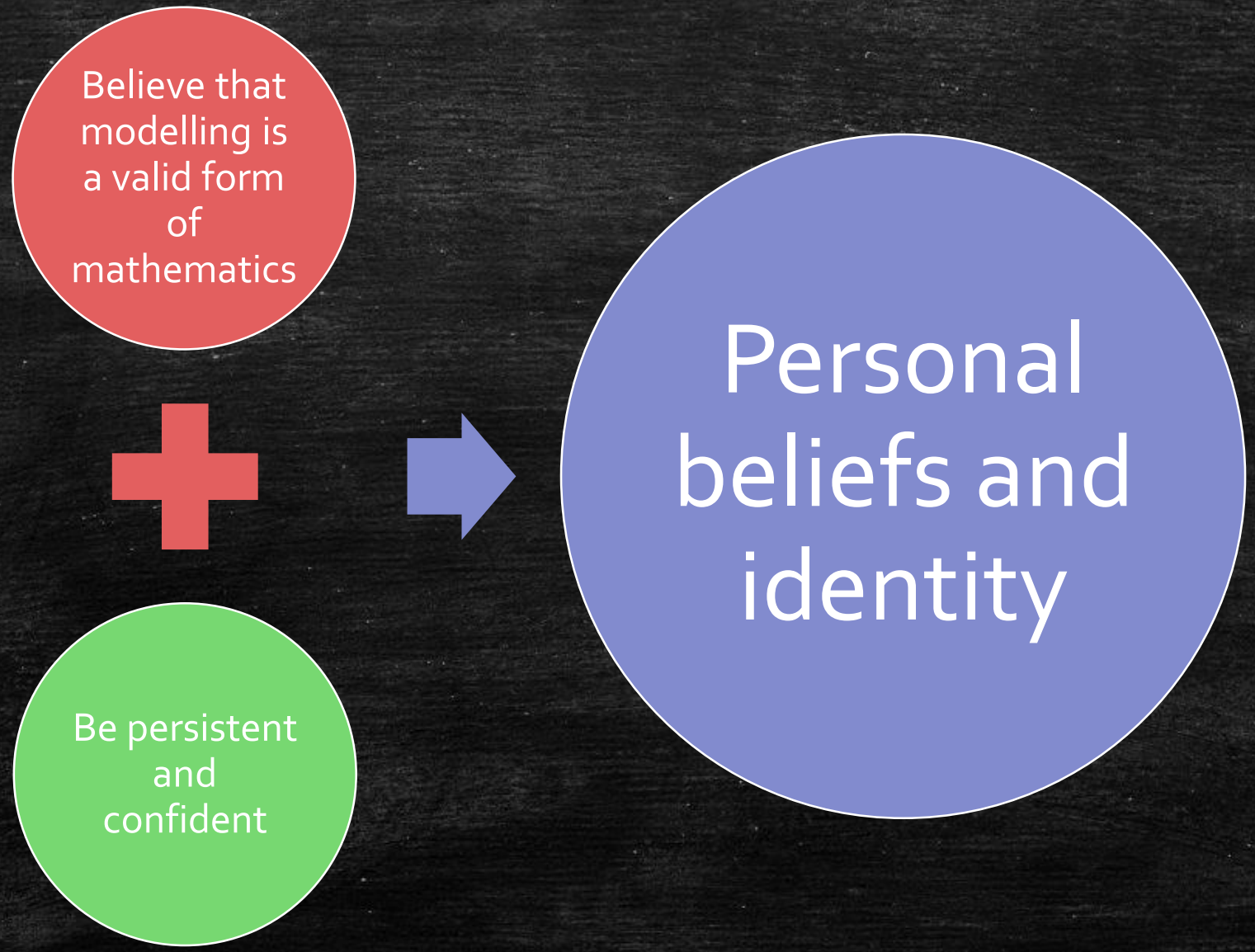
(Weick, 1995 p. xii)

Sensemaking is grounded in identity construction

http://www.africancraftsmarket.com/african-mask-information.html

Van Oers (1998) – perspective of Activity Theory

Niss, 2010 – Enablers for anticipated implementation of modelling



Daher (2015): Positionings and emotions while modelling are affected by

Individual characteristics

History of learning experiences

Activity characteristics

Modelling phases

Brown & Stillman, 2017

Modelling is an ideal environment for students to develop a "sense of mathematics as a way of thinking about life"

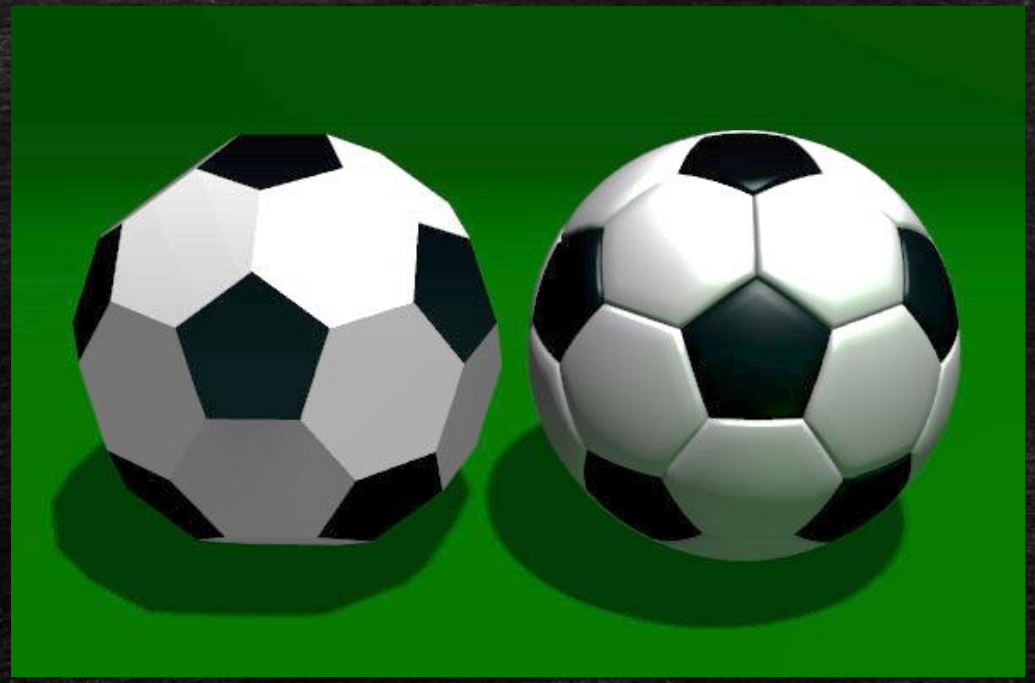


Image: Wikipedia

Sensemaking is retrospective

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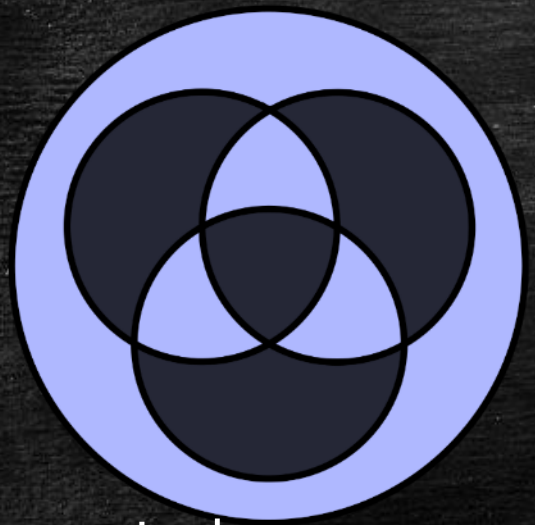
Remembering and looking back are a primary source of meaning making
(Weick, 2001)

Blum & Borromeo Ferri (2016) - individual solutions should be encouraged as they are a basis for reflection



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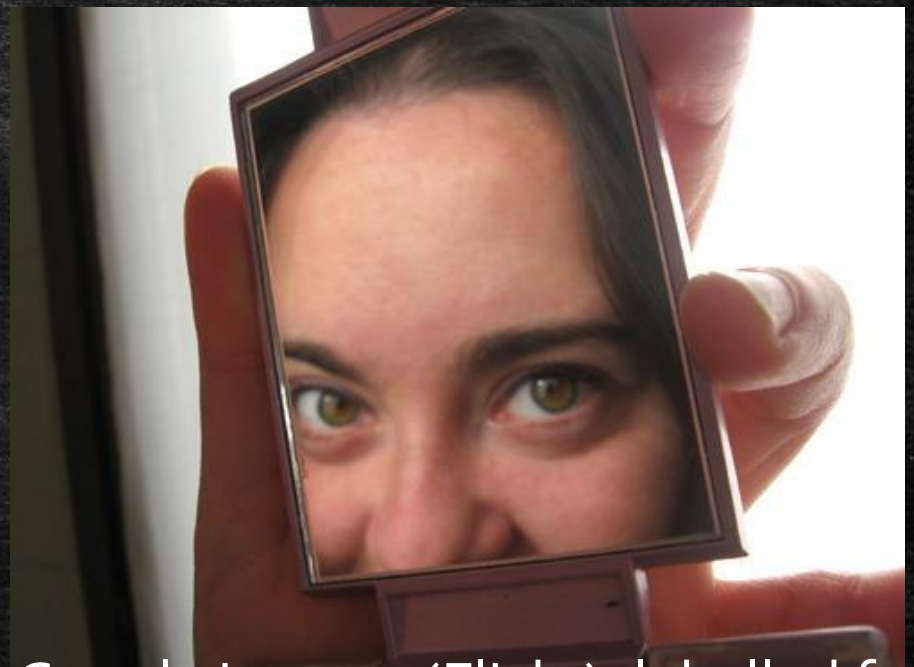

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Brown (2013) - model-eliciting tasks that enable reflection can assist students in understanding that they play an import role in the sensemaking in mathematics.

Sensemaking involves enactment

Weick: by acting, people create materials/objects to project back into the environment. These objects can constrain or enhance sensemaking.

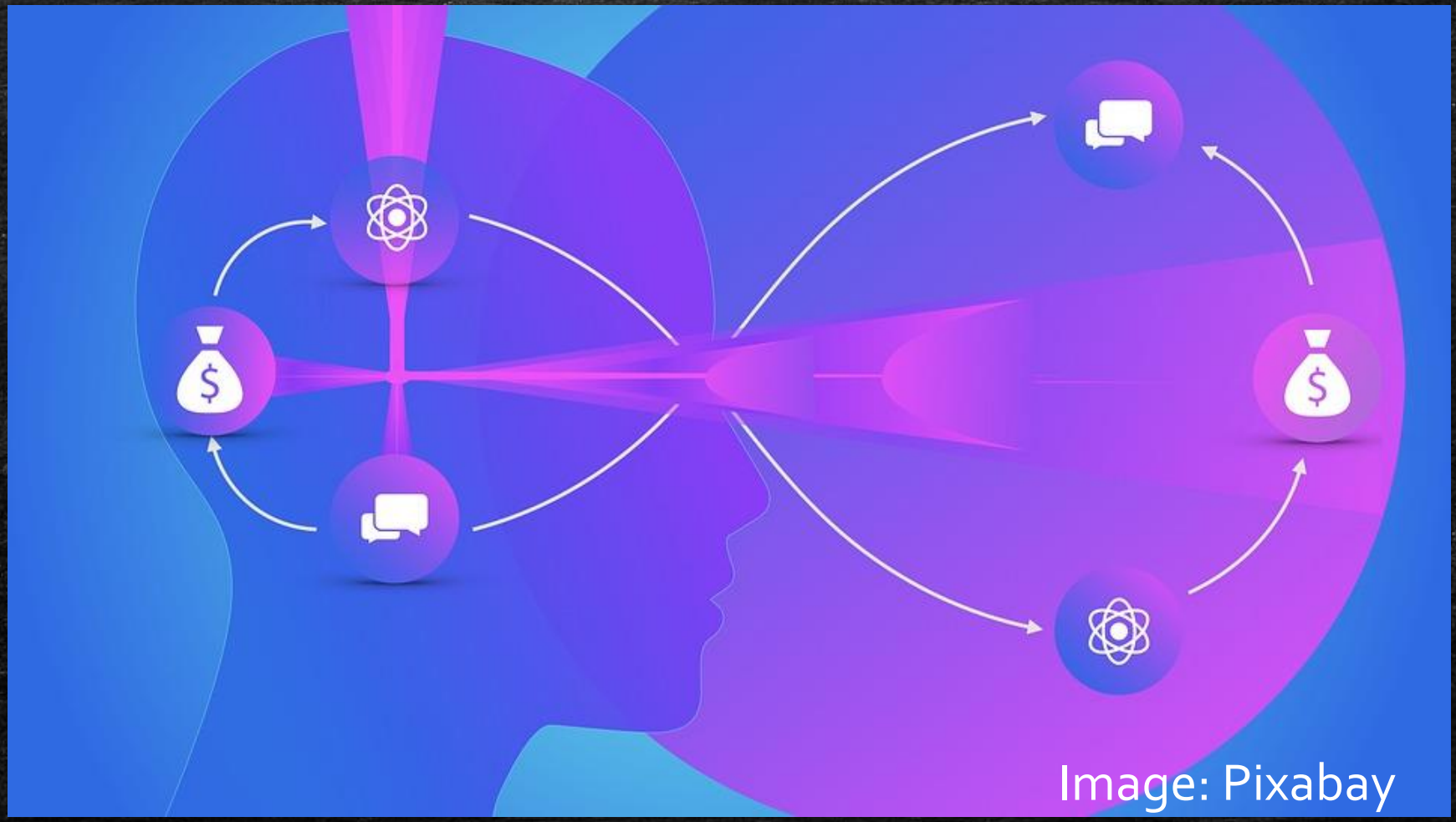
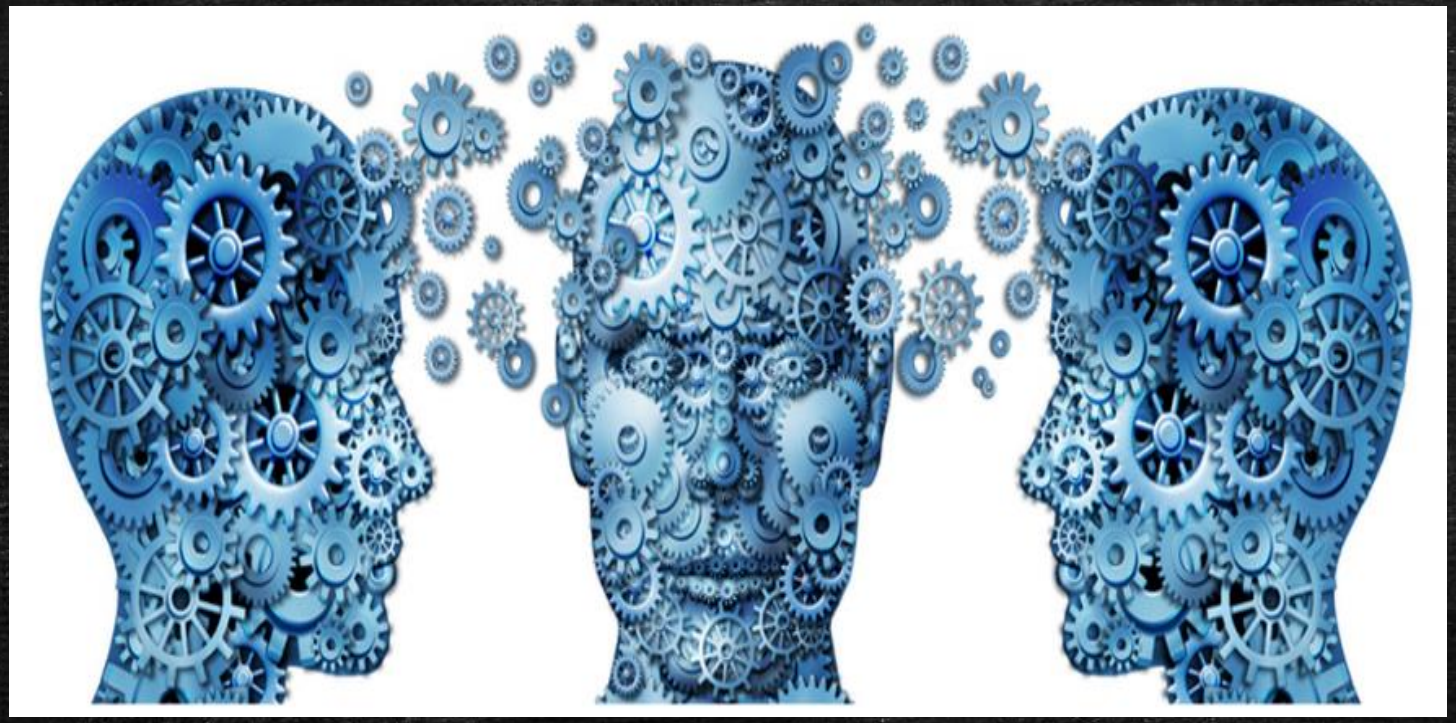


Image: Pixabay

Sensemaking is a social process

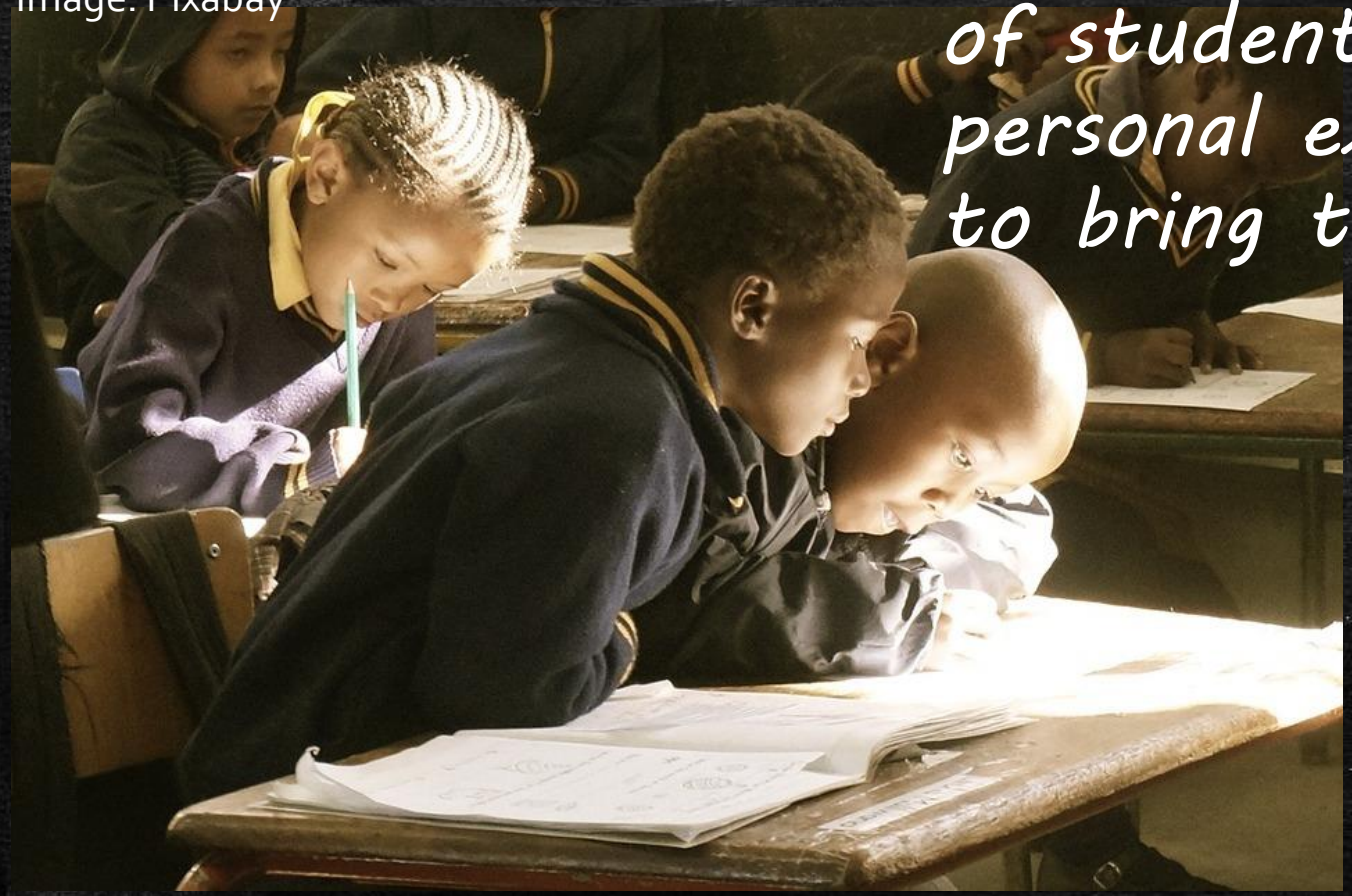


Pic credit: http://www.elearningnetwork.org/wp-content/uploads/2015/03/social-learning-technologies.png

Weick, 2001, p. 178
" A perceived lack of individual capability can often be offset by increased collective capability that heightens collective sensing
and sensemaking"

Brown, 2013 (p. 304)
Novice modellers "lack experience of shared mathematising and negotiation of understanding and meanings contributed to the lack of student beliefs that they had personal experience and knowledge to bring to the solving of the tasks"

Image: Pixabay



Sensemaking is ongoing

Lesh and Harel (2003) constructs are at some intermediate phase of development

Levels vary across tasks and across time within a single task

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Sensemaking is focused on and by extracted cues

Maxpixel:freegreatpicture.com

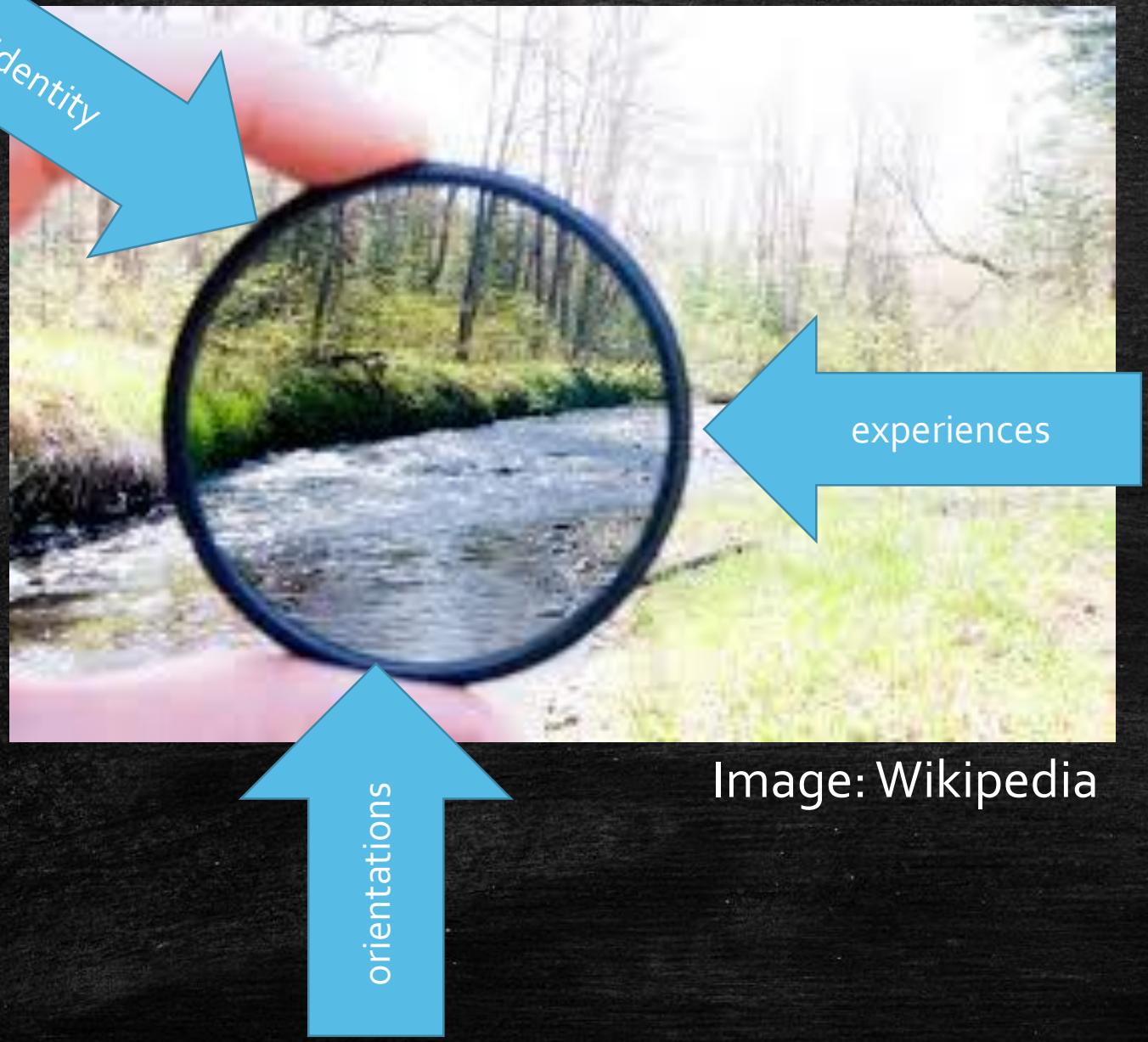


Image: Wikipedia

Schoenfeld 2011.
How we think

Image - Pixabay

Brown and Stillman (2017)

Group member suggestions provide *cues* for other group members

"putting talk to work" or "using talk to think"

Interthinking (Mercer)

Students are drawn into their own and each other's understanding

Andersson, 2010. Sensegiving can provide others with new frames of references...

...since people add interpretations and their own perspectives to the problem.

sensegiving also alludes to students being allowed to make choices

Schatz & Bartlett, 2013

Intuition - integration of important cues

Intuitive sensemaking

"a conscious process informed by subconscious intuitive mechanisms and moderated by deliberate metacognitive effort, with the intention of understanding connections, interpreting meaning, and anticipating trajectories, which support later decision making and possible actions".

Sensemaking is about plausibility before accuracy

Image: Pixabay

Modelling solutions often "fit well, but not perfectly, their constraints" and the mathematical systems that students develop are not "fully stable or comprehensive" (Hamilton, 2007 p. 5)

Plausible reasoning is fluid while demonstrative reasoning follows strict rules

Polya, 1954:

Certainly, let us learn proving, but
also let us learn guessing

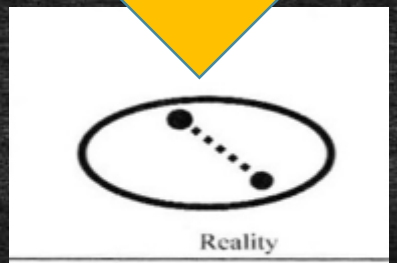
identity

social process

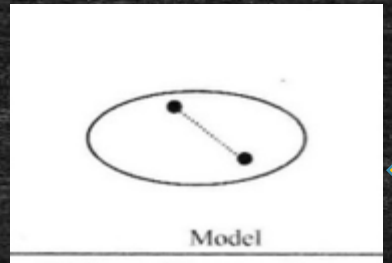
cues

enactment

(a) Modelers bring to bear their prior experiences that they use to interpret the real situation



(b) Modelers will notice some patterns and some details in the real situation…



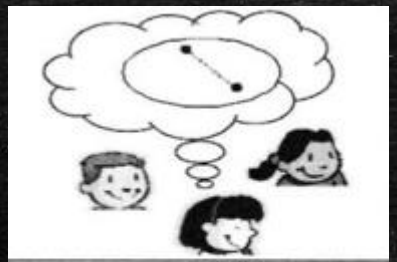
(c) They will express these patterns and details in a model of the real situation

Yoon and Thompson, 2007

retrospection

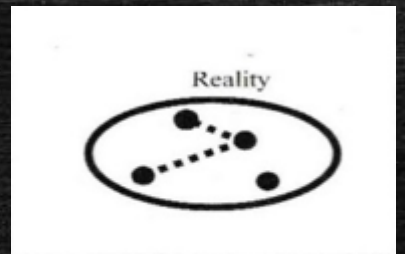
social process

ongoing



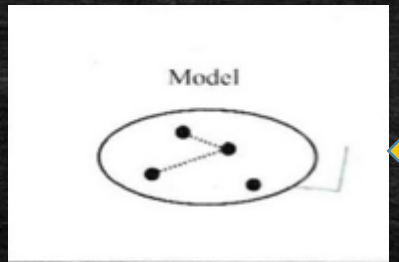
(d) Modelers will then re-interpret the real situation using this model…

cues



(e) This will enable them to notice *new* patterns and details that may conflict with their model

ongoing



(f) They will express these new patterns and details in a revised model of the real situati

enactment

retrospection

Conclusion

Boaler (2014)

We need to give students mathematics problems that allow them space to learn

space for choices

space to make decisions

space for elements of sensemaking

Thank you

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